

Week 8 - Wednesday

COMP 2230

Last time

- Exam 2
- Before that:
 - Equivalence relations
 - Partial order relations

Questions?

Assignment 4

Logical warmup

- Each of 100 baskets contains some number (could be zero) of apples and some number (could also be zero) of bananas
- Show that you can collect 51 of those baskets that together contain at least half the apples and at least half the bananas
- It's actually possible to extend this to a third fruit and still use only 51 baskets (but the math is more confusing)

Partial Orders

Antisymmetry

- Let R be a relation on a set A
- R is **antisymmetric** iff for all a and b in A , if $a R b$ and $b R a$, then $a = b$
- That is, if two different elements are related to each other, then the relation is **not** antisymmetric
- Let R be the "divides" relation on the set of all positive integers
- Is R antisymmetric?
- Let S be the "divides" relation on the set of all integers
- Is S antisymmetric?

Partial orders

- A relation that is reflexive, antisymmetric, and transitive is called a **partial order**
- The subset relation is a partial order
 - Show it's reflexive
 - Show it's antisymmetric
 - Show it's transitive
- The less than or equal to relation is a partial order
 - Show it's reflexive
 - Show it's antisymmetric
 - Show it's transitive

Hasse Diagrams

- Let set $A = \{1, 2, 3, 9, 18\}$
- Let R be the "divides" relation on A
- Draw A as a set of points and connect each pair of points with arrows if they are related with R
- Now, delete all loops and transitive arrows
- This is a **Hasse Diagram**

Total orders

- Let R be a partial order on set A
- Elements $a, b \in A$ are **comparable** if either $a R b$ or $b R a$ (or both)
- If all the elements in a partial order are comparable, then the partial order is a total order
- Let R be the "less than or equal to" relation on \mathbb{R}
 - Is it a total order?
- Let S be the "divides" relation on positive integers
 - Is it a total order?

Probability

Definitions

- A **sample space** is the set of all possible outcomes
- An **event** is a subset of the sample space
- Formula for equally likely probabilities:
 - Let S be a finite sample space in which all outcomes are equally likely and E is an event in S
 - Let $N(X)$ be the number of elements in set X
 - Many people use the notation $|X|$ instead
 - The probability of E is $P(E) = \frac{N(E)}{N(S)}$

Card examples

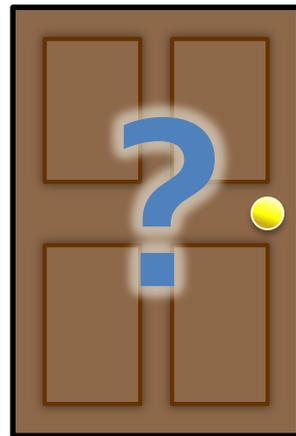
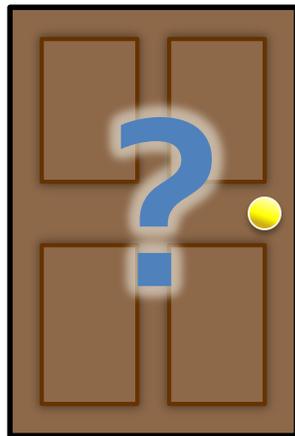
- There are 52 cards in a normal Anglo-American deck of cards
 - Four suits: **Spades**, **Hearts**, **Clubs**, and **Diamonds**
 - 13 denominations: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King
- Imagine you draw a single card from a thoroughly shuffled deck
 - What is the sample space?
 - What is the event of drawing a black face card?
 - What is the probability of drawing a black face card?

Dice example

- Six-sided dice have, uh, six sides, numbered 1 through 6
- If you roll two dice
 - What is the sample space of outcomes?
 - What is the event that the two dice add up to 7?
 - What is the probability that the two dice add up to 7?
 - What about all the other possible values?

Monty Hall

- Imagine you are playing a game show with 3 doors
- There is a prize behind one and nothing behind the other two
- As the contestant, you pick a door, but it isn't opened yet
- The host Monty Hall opens one of the other two doors, revealing nothing
- Then, you get a chance to switch
- Should you stay or switch or does it matter?



Counting the elements in a list

- As a computer scientist, you have almost certainly figured this out
- But, to formalize it, if you have a list numbered m through n , with no elements missing, the total number of elements are $n - m + 1$
- For example, there are 50 elements in an array indexed from 0 to 49
- Likewise, there are 39 elements in a list numbered from 67 to 105

Ticket Out the Door

Upcoming

Next time...

- Multiplication rule
- Addition rule
- Pigeonhole principle

Reminders

- **Start Assignment 4**
- Read 9.2, 9.3, and 9.4